

STATISTICAL INFERENCE



LIKELY?

WHY?

- variance is low.
- seem very separate.



maybe more.



not likely

How much variance is there?

Where is the centre of the data?

VARIATION.

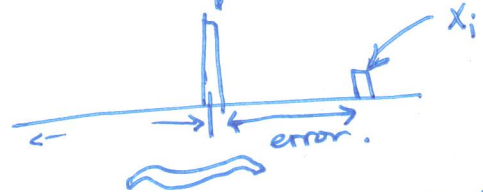
Tell a class: measure this worm's length.



student	cm.
1	2.1
2	2.2
3	2.0
4	2.1
5	1.9
6	2.2
7	2.0
8	2.1

actual length:
probably
about \bar{x}

hope but not necessarily estimate.



* assumption: error is "normal"
→ evenly distributed on either side, & follows the usual distribution.

~~true mean~~

$$\begin{aligned}
 X_{\text{kenny}} &= \text{worm} + E_{\text{kenny}} \\
 &\vdots \\
 X_{\text{bobby}} &= \text{worm} + E_{\text{bobby}}
 \end{aligned}$$

\bar{X} true mean population
↑
estimated by \bar{x}
standard error of the mean = $\frac{s}{\sqrt{n}}$
basic idea: more samples mean closer estimate.

Simple Experiment

3/

Compare the perf using text prediction or not.

H₀: perf is same.

Between subjects

Within subjects.
-balance ordering!

	text prediction	no text predict.
P ₁	231	
P ₂	245	
P ₃	236	
P ₄	241	
P ₅		261
P ₆		253
P ₇		255
P ₈		241

	text p.	no text p.
P ₁	231	261
P ₂	245	255
P ₃	236	241
P ₄	241	241

~~INSERT #5~~

THOUGHT EXPERIMENT:

Where does variance come from?

▷ TO 4.5

$$x_i = \bar{X} + \underbrace{\left(\text{effect of the condition} \right)}_{\text{deviation}} + \epsilon$$

↑ grand mean.
↑ measurement or individual variation

~~$x_i = \bar{X} + \text{text prediction} + \epsilon_i$~~
 ~~$x_i = \bar{X} + \text{no text prediction} + \epsilon_i$~~

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{X_1, X_2} \cdot \sqrt{\frac{2}{n}}}$$

↑ pooled standard deviation.

$$S = \sqrt{\frac{\sum(\bar{X} - x_i)^2}{n}}$$

$$S_{X_1, X_2} = \sqrt{\frac{S_{X_1}^2 + S_{X_2}^2}{2}}$$

If t is large, the relative size of group differences is bigger than measurement (variance) ~~within~~ error.

WORKING IT:

$$t = \frac{\bar{X}_{\text{test prediction}} - \bar{X}_{\text{no test predict}}}{S_{X_1, X_2} \cdot \sqrt{\frac{2}{n}}}$$

→ value →

compare this to a t_{critical}

In this case, it is $n-2$.

↖ based on "degrees of freedom"
↖ α value.
↖ practically, this is related to # of participants

~~WHERE DOES THIS VARIANCE COME FROM?~~

4/ ~~4.5~~

SIDEBAR:

differences among different groups of subjects

↓ differences among subjects in the same groups.

If H_0 is true,

this reduces to

$$\frac{\text{experimental error}}{\text{experimental error}}$$

= 1 ^{over time.}

If H_0 is false:

Something else going on in the numerator!

$$\frac{(\text{treatment effect}) + (\text{experimental error})}{\text{experimental error}}$$

= overtime > 1.

* on a given experiment, it is possible to obtain this effect!

4.5 / ~~WHERE DOES VARIATION COME FROM~~

Deviation

①/

$$\text{Total Deviations} = \left(\text{deviation due to the condition} \right) + \left(\text{deviation due to individual} \right) + \epsilon.$$

practically, we simply roll these terms together

$$\text{total deviation} = \text{between groups deviation} + \text{within group deviation}$$

~~Intuitively, If there is no deviation due to the group condition, then (total deviation = within group variance)~~

~~Another way to see this~~

$$\frac{\text{total deviation}}{\text{within groups deviation}} = \frac{\text{between groups deviation} + \text{within groups deviation}}{\text{within groups deviation}}$$

~~If H₀ is true (i.e. between groups deviation)~~

~~⇒ ratio = 1.~~

~~otherwise:~~

$$\frac{bgd + wgd}{wgd}$$

⇒ (treatment effect + experimental variation)
↑ experimental variation.

the bigger this is,
the bigger the overall value

~~Devations are related to variance~~
~~Sum of sum of squares.~~

6/

~~Devations~~

Sums of squares

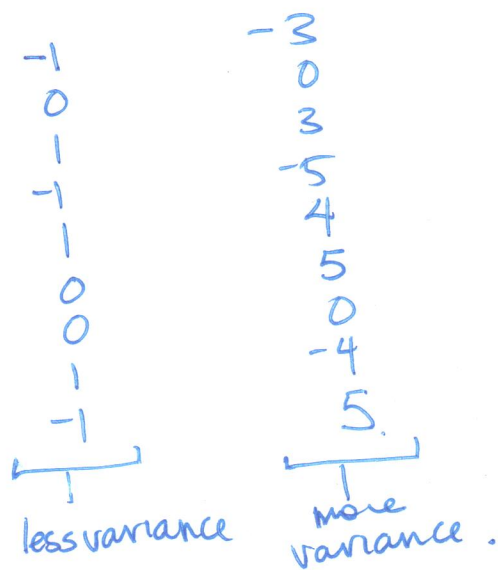
Sum of the squared deviations from the mean.

deviation = $(\bar{X} - x_i)$

this variance is captured better
 w squared deviations: $(\bar{X} - x_i)^2$.

▷ INSERT 7.5

$SS_{TOTAL} = SS_{BETWEEN} + SS_{WITHIN}$.



$SS_{TOTAL} = SS_{TREATMENT} + SS_{ERROR}$.

$SS_{TOTAL} = \sum (x_i - \text{grand mean})^2$.

$SS_{TREATMENT} = n(\text{mean of condition 1} - \text{grand mean})^2 + n(\text{mean of condition 2} - \text{grand mean})^2 + \dots$

$SS_{ERROR} = \sum (x_i - \text{mean of condition 1})^2 + \sum (x_i - \text{mean of condition 2})^2 + \dots$

F-test (one-way ANOVA)

7/

Source	SS	df.	MS	F
between.	SS_{BETWEEN}	$a-1$	$\frac{SS_{\text{between}}}{df_{\text{between}}}$	$\frac{MS_{\text{between}}}{MS_{\text{within}}}$
within	SS_{WITHIN}	$a(n-1)$	$\frac{SS_{\text{within}}}{df_{\text{within}}}$	
total.	SS_{TOTAL}	$an-1$		

F-value, compare to F_{crit}

R will output a p-value too

7.5/ Speed of text entry using: normal typing, iPhone text prediction, Android text predic

H_0 = all the means are equal
 IV = text-typing condition
 DV = Speed (s).

normal	iPhone	Android
10.0	8.8	9.3
9.7	8.6	9.2
8.5	7.0	12.2
10.6	12.0	9.0
10.2	9.1	8.1
9.9	9.2	9.9
avg?		
grand avg	$\frac{4}{3}$	